Quantifying *Variability* & *Uncertainty in Phosphorus TMDL's for Lakes*

*Reservoirs Too!*

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Quantifying Variability & Uncertainty

Benefits

Estimates Probability of Achieving Goal

Provides Rational Basis for MOS

Helps to Define Lake Goal

Numerical Value "Target" or "Limit"?

Spatial & Temporal Averaging

Compliance Rate (% of Years Achieving Goal)

Identifies Important Sources of Uncertainty

Provides Incentive for Continued Data Collection & Modeling

More Data --?--> Lower MOS --?--> Higher Load Alloc
Quantifying Variability & Uncertainty Difficulties

Limited Guidance Provided in TMDL Regulations

Frequency Concepts Rarely Built into WQ Standards

Load Allocations Sensitive to Assumptions:

  Compliance Rate (e.g., % of yrs <= target)

  Confidence Level (~probability of success)

Uncertainty/Variability Costs (MOS) Can Be Large

Can Backfire & Retard Restoration Efforts

Technical Complexity

Uncertainty Estimates are Uncertain
Bloom Frequencies from daily samples at Vadnais Intake & Pleasant Gatehouse
Total Phosphorus concentrations measured in Lake Epilimnion (0-6 m)
April-September Means for Each Year

Algal Bloom Frequency vs. Total Phosphorus

Bloom Frequencies from daily samples at Vadnais Intake & Pleasant Gatehouse
Total Phosphorus concentrations measured in Lake Epilimnion (0-6 m)
April-September Means for Each Year
Derivation of Phosphorus Target for Upper Klamath Lake for Compliance with pH Standard

Yearly Means by Lake Region, April-October

Frequency = % of Measurements (All Stations & Depths) Exceeding pH 9

Historical Range

Target

Lake Mean Total P (ppb)
Case 1: Long-Term Mean < 20 ppb
TMDL = 100 kg/yr

Case 2: Yearly Mean < 20 ppb in 90% of Years
Long-Term Mean = 15.5 ppb
TMDL = 77 kg/yr
TMDL Equations
Long-Term-Average Mass Balances

Watershed Mass Balance:

\[ \text{TMDL} = \sum \text{LAs} + \sum \text{WLAs} + \text{Background} + \text{MOS} \]

- Total Maximum Daily Load
- Non-Point Sources
- Point Sources
- Natural or Unregulated
- Margin of Safety
- Anthropogenic
- < Discharge Permit
- Undev. Watershed
- Atmospheric
- uncertainty
- variability

\(--\text{ Expected Long-Term-Average Load to Lake }\)-->

Lake Mass Balance:

\[ \text{TMDL} = Q_S P^* + U P^* \]

- Input
- Flushing
- Net Retention
Consideration of Point-Source Variability

Arith Mean 10 Long-Term Average Load Used in TMDL Mass Balance
Permit Limit 14 Permit Value not to be Exceeded in >5% of Months
Model: Log-Normal Distribution with CV = 0.2
MOS Alternatives

Conservative Water Quality Criteria/Standard
Conservative Phosphorus Goal
Conservative Modeling Assumptions
Conservative Effluent Limits / Discharge Permits
Conservative Facility Designs
Conservative Growth Projections
Shell Game
Modeling Variability & Uncertainty
Stochastic Approach

Predicted Long-Term-Average Lake P Conc:

\[ P_M = \frac{L_M}{U + Q_S} \]

Accounting for Uncertainty:

\[ P_{MU} = P_M \exp(\delta_u) \]
\[ \delta_u = \text{random error term, mean} = 0, \ \text{std dev} = \sigma_u \]
\[ \sigma_u \sim 0.1 - 0.5 \]

Accounting for Uncertainty & Variability:

\[ P_{MUY} = P_M \exp(\delta_u + \delta_y) \]
\[ \delta_y = \text{random yr-to-yr variation, mean} = 0, \ \text{std dev} = \sigma_y \]
\[ \sigma_y \sim 0.1 - 0.3 \]
<table>
<thead>
<tr>
<th>Variable</th>
<th>Units</th>
<th>Value</th>
<th>Equation</th>
<th>Notes</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Input Values:</strong></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Existing Load</td>
<td>mg/m²-yr</td>
<td>1000</td>
<td>Lo</td>
<td>long-term-average load</td>
</tr>
<tr>
<td>Net Settling Rate</td>
<td>m/yr</td>
<td>10</td>
<td>U</td>
<td>from model calibration</td>
</tr>
<tr>
<td>Water Load</td>
<td>m/yr</td>
<td>10</td>
<td>Qs</td>
<td>outflow / surface area</td>
</tr>
<tr>
<td>Target Lake P</td>
<td>ppb</td>
<td>25</td>
<td>P*</td>
<td>for compliance with wq standards</td>
</tr>
<tr>
<td>Confidence Level</td>
<td>%</td>
<td>90%</td>
<td>p1</td>
<td>= 100 - max risk of not achieving objective</td>
</tr>
<tr>
<td>Compliance Frequency</td>
<td>%</td>
<td>80%</td>
<td>p2</td>
<td>expected percent of years achieving target</td>
</tr>
<tr>
<td>Model Error Std Dev</td>
<td>-</td>
<td>0.3</td>
<td>Su</td>
<td>accounts for modelling uncertainty</td>
</tr>
<tr>
<td>Year-to-Year Std Dev</td>
<td>-</td>
<td>0.1</td>
<td>Sy</td>
<td>accounts for temporal variability in lake p</td>
</tr>
<tr>
<td><strong>Output Values:</strong></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Normal Deviate (p1)</td>
<td></td>
<td>1.282</td>
<td>Zu = Normal (1-p1)</td>
<td>normal deviate with tail probability 1-p1</td>
</tr>
<tr>
<td>Uncertainty Factor</td>
<td></td>
<td>0.681</td>
<td>Fu = exp (- Zu Su)</td>
<td></td>
</tr>
<tr>
<td>Normal Deviate (p2)</td>
<td></td>
<td>0.842</td>
<td>Zy = Normal (1-p2)</td>
<td>normal deviate with tail probability 1-p2</td>
</tr>
<tr>
<td>Variability Factor</td>
<td></td>
<td>0.919</td>
<td>Fy = exp (- Zy Sy)</td>
<td></td>
</tr>
<tr>
<td>MOU Fraction</td>
<td></td>
<td>0.8</td>
<td>f = (1 - Fu) / (2 - Fu - Fy)</td>
<td>fraction of MOS assigned to MOU</td>
</tr>
<tr>
<td>TMDL</td>
<td>mg/m²-yr</td>
<td>500</td>
<td>TMDL = (Qs + U) P*</td>
<td>long-term-average load</td>
</tr>
<tr>
<td>Allocated Load</td>
<td>mg/m²-yr</td>
<td>313</td>
<td>La = TMDL Fu Fy</td>
<td>long-term-average allocated load</td>
</tr>
<tr>
<td>Margin of Safety</td>
<td>mg/m²-yr</td>
<td>187</td>
<td>MOS = TMDL - La</td>
<td>or MOS = MOU + MOV</td>
</tr>
<tr>
<td>Margin of Uncertainty</td>
<td>mg/m²-yr</td>
<td>149</td>
<td>MOU = f MOS</td>
<td>portion of MOS attributed to uncertainty</td>
</tr>
<tr>
<td>Margin of Variability</td>
<td>mg/m²-yr</td>
<td>38</td>
<td>MOV = MOS - MOU</td>
<td>portion of MOS attributed to variability</td>
</tr>
<tr>
<td>Uncertainty Cost</td>
<td></td>
<td>30%</td>
<td>MOU / TMDL</td>
<td>MOU as fraction of TMDL</td>
</tr>
<tr>
<td>Variability Cost</td>
<td></td>
<td>8%</td>
<td>MOV / TMDL</td>
<td>MOV as fraction of TMDL</td>
</tr>
<tr>
<td>Required Load Reduction</td>
<td></td>
<td>69%</td>
<td>1 - La / Lo</td>
<td></td>
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</table>
TMDL Sensitivity to Compliance Frequency & Confidence Level

<table>
<thead>
<tr>
<th>Lake P Target</th>
<th>LT-Avg</th>
<th>LT-Avg</th>
<th>10-Yr Max</th>
<th>10-Yr Max</th>
</tr>
</thead>
<tbody>
<tr>
<td>Uncertainty Considered</td>
<td>No</td>
<td>Yes</td>
<td>No</td>
<td>Yes</td>
</tr>
<tr>
<td>Variability Considered</td>
<td>No</td>
<td>No</td>
<td>Yes</td>
<td>Yes</td>
</tr>
<tr>
<td>Confidence Level --&gt;MOU</td>
<td>50%</td>
<td>90%</td>
<td>50%</td>
<td>90%</td>
</tr>
<tr>
<td>Compliance Freq --&gt; MOV</td>
<td>50%</td>
<td>50%</td>
<td>90%</td>
<td>90%</td>
</tr>
<tr>
<td>Model Error Std Dev</td>
<td>0.4</td>
<td>0.4</td>
<td>0.4</td>
<td>0.4</td>
</tr>
<tr>
<td>Temporal Std Dev</td>
<td>0.2</td>
<td>0.2</td>
<td>0.2</td>
<td>0.2</td>
</tr>
<tr>
<td>Allocated LTA Load</td>
<td>500</td>
<td>299</td>
<td>387</td>
<td>232</td>
</tr>
<tr>
<td>Load Reduction</td>
<td>29%</td>
<td>57%</td>
<td>45%</td>
<td>67%</td>
</tr>
</tbody>
</table>
TMDL Sensitivity to Model Uncertainty

- Increasing Model Uncertainty

<table>
<thead>
<tr>
<th>Metric</th>
<th>0.0</th>
<th>0.1</th>
<th>0.2</th>
<th>0.4</th>
</tr>
</thead>
<tbody>
<tr>
<td>Model Error Std Dev</td>
<td></td>
<td></td>
<td></td>
<td></td>
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<tr>
<td>Temporal Std Dev</td>
<td>0.2</td>
<td>0.2</td>
<td>0.2</td>
<td>0.2</td>
</tr>
<tr>
<td>Allocated LTA Load</td>
<td>387</td>
<td>340</td>
<td>299</td>
<td>232</td>
</tr>
<tr>
<td>Load Reduction</td>
<td>45%</td>
<td>51%</td>
<td>57%</td>
<td>67%</td>
</tr>
<tr>
<td>Uncertainty Cost</td>
<td>0%</td>
<td>11%</td>
<td>20%</td>
<td>34%</td>
</tr>
</tbody>
</table>

TMDL Allocations for Confidence Level = 90%, Compliance Freq = 90%
Figure 29
Confidence Intervals for TMDL's Compared with Historical Phosphorus Loads
Iterative TMDL Process

- Define Management/Use Objectives
- Identify Water Quality Standards
- Monitor & Model
- Estimate TMDL's
- Evaluate Controls
- Develop Load Allocations
- Implement Controls
- Monitor Results
- Objectives Achieved?

~ 5-10 years?
Confidence Level  
50%  90%  90%

Compliance Freq  
50%  90%  90%

Model Error Std Dev  
0.4  0.4  0.1

Temporal Std Dev  
0.2  0.2  0.2

Allocated Load  
500  232  340

Cum Load Reduction  
50%  77%  66%

Phased Approach to TMDL Implementation

- Phase 1: Implementation & Data Collection 5-10 yrs?
- Phase 2:

Graph showing Total P Load (mg/m²/yr) for Current, TMDL-1, Alt, Current, and TMDL-2 with legend:
- Existing
- MOU
- MOV
- Allocated

5-10 yrs?